I Semester B.C.A. Degree Examination, December 2018  
(CBCS Scheme)  
COMPUTER SCIENCE  
Discrete Mathematics  

Time : 3 Hours  
Max. Marks : 100  

Instruction : Answer all Sections.  

SECTION – A  

I. Answer any ten of the following.  

(10×2=20)  

1) Write the following sets in set-builder form  
   a) \( \{2, 5, 8, 11, \ldots\} \)  
   b) \( \{1, 4, 9, \ldots, 100\} \)

2) Define universal set. Give an example.  

3) Let \( A = \{1, 2, 3, 4, 6, \ldots\} \). Let \( R \) be the relation defined by \( R = \{(a, b) \mid a \in A, b \in A \)  
   a) \( a \) divides \( b \).  
   b) Write the elements of \( R \).  
   c) Write the domain of \( R \).  

4) Define Tautology.  

5) What is upper triangular matrix? Give an example.  

6) Find the value of \( x \).  
   a) \( \log_8 64 = x \)  
   b) \( \log_2 27 = 3 \)  

7) How many different signals can be made by 6 flags of different colors?  

8) Define a group.  

9) \( \bar{a} = 3i - 4j, \bar{b} = 2i + j \), find \( |\bar{a} + \bar{b}| \).  

10) Find the value of \( 'a' \) if the distance between the points \( (a, 2) \) and \( (3, 4) \) is \( \sqrt{8} \) units.  

11) If the centroid of the triangle ABC is \( (2, 3) \) and \( A = (4, 2) \) and \( B = (4, 5) \). Find the co-ordinates of \( C \).  

12) Define slope of a line.
23) A examination question paper consists of 12 questions divided into Part A and Part B. Part A consists of 7 questions and Part B consists of 5 questions. In how many ways can a student answer 8 questions if
   a) there is no condition put in the paper
   b) the student has to answer 5 from Part A and 3 from Part B.

24) Show that \((\mathbb{Z}_n, +_n)\) where \(\mathbb{Z}_5 = \{0, 1, 2, 3, 4, 5\}\) is a group.

25) Show that the set of all fourth roots of unity form a group under multiplication.

26) Show that the points with position vector \(2i - j + k, i - 3j - 5k\) and \(3i - 4j - 4k\) are the vertices of a right angled triangle. Also find the remaining angles of the triangle.

27) Show that the points \(A (2, 3, -1), B (1, -2, 3), C (3, 4, -2)\) and \(D (1, -6, 6)\) are coplanar.

28) Find the area of the parallelogram whose diagonals are
   \[ \vec{a} = 3i + j - 2k \] and \[ \vec{b} = i - 3j + 4k. \]

SECTION - D

IV. Answer any four of the following.

29) Show that the points \((2, -1), (3, 4), (-2, 3)\) and \((-3, -2)\) form a rhombus.

30) Find the area of the quadrilateral whose vertices are \((1, -1), (7, -3), (12, 2)\) and \((7, 21)\).

31) Find the equation of the locus of point which moves such that it is equidistant from the points \((1, 2)\) and \((-2, 3)\).

32) Show that the line joining the points \((2, 3)\) and \((4, 2)\) is perpendicular to the line joining the points \((5, 3)\) and \((6, 5)\).

33) Find the equation of the line passing through \((5, -2)\) and making an angle 150° with x-axis in the positive direction.

34) Find the equation of the line passing through \((-2, 6)\) and sum of the intercepts on the co-ordinate axes is 5.
SECTION – B

II. Answer any six of the following. (6x5=30)

13) If \( U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \) is the universal set. \( A = \{2, 3, 4, 8\} \), \( B = \{1, 3, 4\} \) and \( C = \{3, 4, 5, 6\} \) verify

\[
(A \cup B)' = A' \cap B' \quad \text{and} \quad (A \cap B)' = A' \cup B'.
\]

14) Let \( A = \{-2, -1, 0, 1, 2\} \), \( B = \{-3, -1, 1, 5\} \). Define \( f : A \rightarrow B \) by \( f(a) = 2a^2 - 3 \), for all \( a \in A \). Is \( f \) one-one? On to? Find \( f^{-1}(5) \) and \( f^{-1}(-1) \).

15) Show that the proposition \((p \land q) \land \neg(p \lor q)\) is a contradiction.

16) Write the converse, inverse and contrapositive of the conditional "If two integers are equal then their squares are equal".

17) Find the inverse of the matrix

\[
\begin{bmatrix}
2 & -1 & 3 \\
-1 & 4 & 2 \\
0 & -3 & 1
\end{bmatrix}
\]

18) Solve using Cramer’s rule

\[
5x + 2x + z = -1; \quad x + 7y - 6z = -18; \quad 3y + 6z = 9.
\]

19) Find the eigenvalues and eigen vectors of the matrix

\[
\begin{bmatrix}
4 & 1 \\
-1 & 2
\end{bmatrix}
\]

20) Verify the Cayley-Hamilton theorem for the matrix \( A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \).

SECTION – C

III. Answer any six of the following. (6x5=30)

21) If \( a^2 + b^2 = 7ab \) S.T.

a) \( 2 \log (a + b) = 2 \log 3 + \log a + \log b \)

b) \( 2 \log (a - b) = \log 5 + \log a + \log b \)

22) In how many ways 3 boys and 5 girls can be arranged in a row so that

a) no two boys together?

b) all girls are together